Frequency Domain Model Identification of Quadrotor at Hover

Marvi Jamali, Dr. Arbab Nighat, Muhammad Sharif Jamali

ABSTRACT- Dynamic characteristics such as intensively nonlinear, multivariable, highly connected system, a quadrotor is regarded as an unstable system, even though it is essential to have a more reliable and reasonable vehicle control model that tends to be easy to use. Modeling is very necessary as model gives the complete narration of how the system complies with the inputs given. Owing to its complex structure, the quad rotor model is not really a simple task. Modeling techniques such as transfer function and system identification method can be used to derive model. Frequency response identification is an inflexible procedure to stimulate accurate aerial vehicle dynamic models from the measured response to control inputs immediately and quickly. This research presents the model's stimulation detail that correlates with a quad rotor's hover operating condition. The Newton - Euler approach is the mathematical approach used for this modeling. Due to its suitability to identify rotary - wing quadrotor dynamics, the Mat Lab system identification toolbox is used.

Keywords- Unmanned Aerial Vehicle, Dynamics of Quadrotor, System Identification toolbox, X-plan.

1. INTRODUCTION

MANY researchers have become aware of quadrotor because the quadrotor application for improvement is enormous [1]. Mostly Quadrotor is used in military application. The UAV 'S (like Quadrotor) has now been widely used for many other applications in agriculture, commerce, science and many others. A quadrotor is a four-rotor-driven multirotor helicopter. The rotors are placed in a square structure equal to the quadrotor's mass center. Each paddle / propeller is connected directly to the brushless motors. The quadrotor has a fixed - pitch blade and the flow of air from each blade downwards generates an upward elevator. Quad rotors also have the ability to take off and land vertically at low altitude. They can also swing while flying [2]. Its movements are based on three rotational sub systems and three translational sub systems; they are three translational motions along the X, Y, and Z-axes, and three rotational motions (θ, ϕ, ψ) around the axis (X, Y, Z), (Roll, Pitch and Yaw). We need such a mathematical model to analyze the dynamics of the system and design a suitable controller. Therefore, we really use such a mathematical model that has to do with motions and responses in a complicated as well as dynamic system. Quad rotor has four control inputs and six degrees of freedom (DOF). The structure of the quad copter should be symmetrical and the body should be rigid.[3]. "System identification is a methodology that is often used to determine the mathematical model by analyzing input signals (such as frequency - sweep input, step input, double input, etc.) and output state measured from the real dynamic system"[4]. Test data is also used for model validation, Practical models must behave in the same or similar way as a real dynamic system. There are several system identification methods that existed approved are: least squares prediction-error method, least squares, and linear regression etc. [5].

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2. LITERATURE REVIEW

Researchers have applied different methodologies to obtain Quadrotor UAV's parameters. A Genetic Algorithm (GA) is a universally optimum method for solving optimization problems such as quad-rotors system. Distributed Genetic Algorithm (DGA) and Real-coded GA are used to determine the unknown parameters[6], [7]. Model identification is carried out using the least square method, maximum probability estimation fuzzy system identification, neural network identification and walvet network identification methods. [8]. Some examples of these identification methods are: ARX Model is an identification method to determine linear systems [9], RBF-ARX Model combine the linear ARX model structure with Radial Basis Function Neural Network (RBF) [10], [11] and Fuzzy Modeling which describes the dynamic characteristics of complex and nonlinear dynamics . Typically, for the Quadrotor helicopter, two modeling strategies are widely used; they are first principle modeling and system identification modeling. First principle modeling requires higher order and complex differential equations, which sometimes become very hard to handle. "Using traditional modeling methods, the aerodynamic, inertial, and structural characteristics of an aircraft or rotorcraft are analyzed to predict a dynamic model"[12]. After this preliminary model is acquired, it is then simulated and compared with flight-test data [12]. On the other hand, the method of deriving the mathematical model of a system based on experimental data from the control inputs and measured outputs of the system is a system identification. System identification process of the system can be carried out in either the frequency domain or the time domain [13]. The most commonly used tools among the different system

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identification techniques which are reported in the literature are Prediction Error Method (PWM) and Comprehensive Identification from Frequency Response Identification (CIFER) [14]. Most of the research work was carried out using complex models on a simulation - based environment and different identification software was used. While this research focused on a simple mathematical model for achieving better results and using the conventional toolbox for system identification, available in MATLAB to avoid complexity. The frequency sweep signal was preferred as the input signal to excite the quadrotor during in the hover, largely due to its realistic recognition in the flight vehicle identification process. The verification result shows the proposed approach is successful potential as a simple and accurate method of identification.

3. DYNAMICS OF QUAD ROTOR

The Newton-Euler formula is adopted to drive the relationships for the Quadrotor dynamics and kinematics based on the following assumptions: [15].

- Structure is symmetrical and rigid
- Mass distribution is assumed constant.
- Assume hovering condition.
- Drag and thrust forces are proportional to the square of the propellers speed.

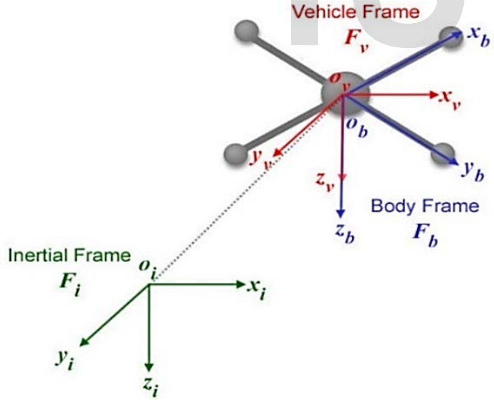

Fig: 3-1 Quadrotor Frames of reference

A Quad rotor's kinematics and dynamics describes how changes take place between coordinate systems. For the study of quadcopter dynamics, the following two coordination systems are adopted:

- The frame of earth inertia (E-frame)
- The vehicle's body-fixed frame (B-frame)

Translation and rotational subsystems are the dynamics of the quadcopter, and both frames are mentioned as references for movement and dynamic equations [15], [16].

The X- axis rotation is rolling (ϕ) and is calculated as follows:

$$R_x(\phi)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\phi & \sin\phi\\ 0 & -\sin\phi & \cos\phi\end{bmatrix} \quad (1)$$

The Y- axis rotation is pitch (θ) and is calculated as follows:

$$R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix} \quad (2)$$

The Z- axis rotation is yaw (ψ) and is calculated as follows:

$$R_z(\psi)=\begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix} \quad (3)$$

In addition, the rotational transfer matrix can be written as:

$$R_B^E = R_x(\phi)R_y(\theta)R_z(\psi) \quad (4)$$

By using the formalism of Newton - Euler, the quadrotor motion equations are given in [17].

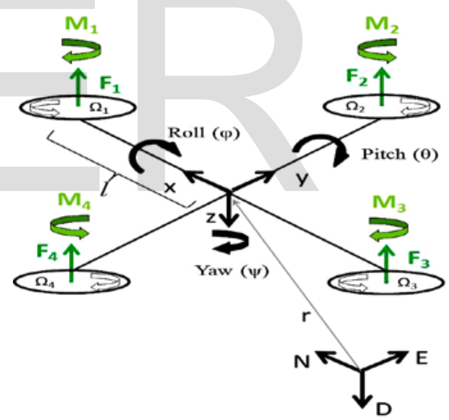

Fig: 3-2. Forces and Movements acting on Quadrotor

Rotational Equation of motion of Quadrotor are:

$$\ddot{\phi}=\dot{\theta}\dot{\phi}\left(\frac{I_{yy}-I_{zz}}{I_{xx}}\right)-\frac{J_r}{I_{xx}}\dot{\theta}\omega+\frac{l}{I_{xx}}U_2$$

$$\ddot{\theta}=\dot{\phi}\dot{\psi}\left(\frac{I_{zz}-I_{xx}}{I_{yy}}\right)+\frac{J_r}{I_{yy}}\dot{\phi}\omega+\frac{l}{I_{yy}}U_3$$

$$\ddot{\psi}=\dot{\phi}\dot{\theta}\left(\frac{I_{xx}-I_{yy}}{I_{zz}}\right)+\frac{l}{I_{zz}}U_4 \quad (5)$$

Translational equation of motion of Quadrotors are:

$$\ddot{X} = \frac{U_1}{m} \times [\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi]$$
$$\ddot{Y} = \frac{U_1}{m} \times [-\cos\psi \sin\phi + \sin\theta \sin\psi \cos\phi] \quad (6)$$
$$\ddot{Z} = \frac{U_1}{m} \times [-g + (\cos\theta \cos\phi)]$$

Where (ϕ) , (θ) and (ψ) are the roll, pitch and yaw angles respectively; I_{xx} , I_{yy} and I_{zz} are the mass moments of inertia in the (X) , (Y) , and (Z) axes respectively; J_r is the rotor inertia; (ω) is the angular velocity of the rotor; l is the length of the rotor arm from the origin of the coordinate system. The following expressions are the four inputs to the four rotors:

$$U_1 = b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right)$$
$$U_2 = bl\left(-\omega_2^2 + \omega_4^2\right)$$
$$U_3 = bl\left(\omega_1^2 - \omega_3^2\right) \quad (7)$$
$$U_4 = d\left(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2\right)$$

Where "U_1", "U_2", "U_3" and "U_4" are the four control inputs, and 'b' and 'd' are thrust and drag coefficients respectively.

4. SIMULATION TOOLS FOR SYSTEM IDENTIFICATION

Using the MATLAB System Identification Toolbox and X-plane flight simulator developed by laminar research, the simulation platform is implemented.

4.1 X-plane Flight simulator

X - Plane flight simulator has been selected as FAA (Federal Aviation Administration) authorizes the preparation of pilots [18], and that can be used as an engineering tool to simulate fixed- and rotary -wing aircraft's flight variable response. The aircraft does not use stability derivatives to describe the flying characteristics of a rotary / fixed wing aircraft as did other simulation engines ; rather than, it uses actual flow calculations to evaluate how the rotary / fixed wing aircraft starts flying in a simulated environment.[19].

Fig: 4-1 X-Plane Flight Simulator

4.2 System Identification toolbox

System identification is a method of extracting from flight data a mathematical description of vehicle or dynamic component behavior. Then these simulation models are used to predict quadrotor movement dynamic behavior. It is used here to identify the quadrotor's transfer function model in hover condition. Fig: 4-2 shows the system identification toolbox.

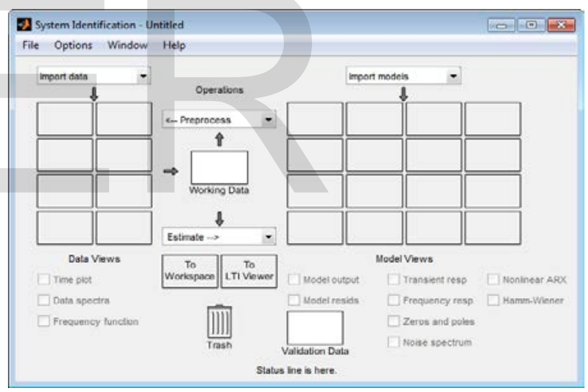

Fig: 4-2 MATLAB System Identification Toolbox's

5. FREQUENCY SWEEP INPUTS AND TIME HISTORY DATA

Quadrotor's identification process begins with that of the compilation of experimental flight data. To make sure the excellent quality of the flight data gathered, it is essential to choose the input signal. A wide range of aircraft system identification frequency excitation signals can be observed in [14]. In this research study, the input signal selected is frequency sweep signal for exciting the quadrotor was selected when hover. The frequency sweep signal has potential of flexibility thus valuable because of signal characteristic, which collects data in many ranges of frequency varying between low to high. This signal frequency

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is used for controlling one surface input of quadrotor during the time meanwhile left over inputs of the system kept not correlated. With reference to [18] the process of identification of rotorcraft needs frequency bandwidth between 0.3 - 12 rad / sec.

After this, the input signal of exciting frequency is written as under:

$$u = A\sin\left[\varphi(t)\right] \tag{8}$$

$$\varphi(t) = \int_0^{2_{rec}} \left[(\omega_{min}) + K(t)(\omega_{min} - \omega_{max})\right] dt \tag{9}$$

$$K(t) = C_2 \left| \exp\left(\frac{C_1 t}{T_{sec}}\right) - 1 \right| \tag{10}$$

The suggested values for C_1=40.0 and $C_2 = 0.0187$ are found in [18] . The Fig: 5-1 shows the typical input of a computer simulated frequency sweep. Equation (8 - 10) are used for computerized frequency sweeps on the quadrotor model.

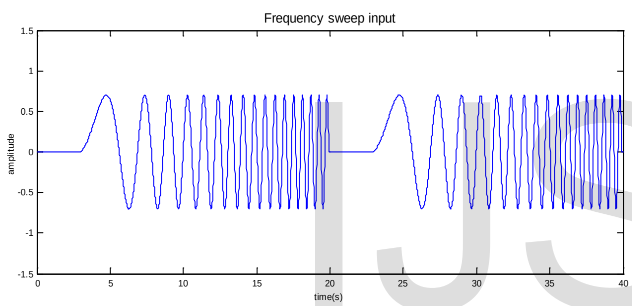

Fig: 5-1 Automated frequency sweep input

Table 5.1 Frequency sweep design specification for model identification

Design Parameters	**Values**	**Units**
Range of frequency	0.3-15	Rad/sec
Period of initial/final trim	0.3	s
Record time	10	s

Table 5.2 provides the system parameters for MATLAB simulations.[15]

Table 5.2 Parameter Values for Simulation

Parameter	**Symbols**	**Values**	**Units**
Mass	m	0.650	m
Acceleration due gravitational	g	9.81	m/s^2
Axel Length	l	0.23	m
Rotor inertia	Jr	$6e^{-5}$	Kg-m^2
Moment of inertia	I_{xx} I_{yy} I_{zz}	$6.228e^{-3}$ $6.228e^{-3}$ $1.12e^{-2}$	kg-m^2
Aerodynamic force and moments	K_f	$3.13e^{-5}$	

6. SYSTEM IDENTIFICATION PROCEDURE FOR QUADROTOR USING SID TOOLBOX

The transfer function is generated using MATLAB system identification toolbox representing the model of a quadrotor for altitude control. Fig: 6-1 shows the MATLAB system identification Toolbox's is used.

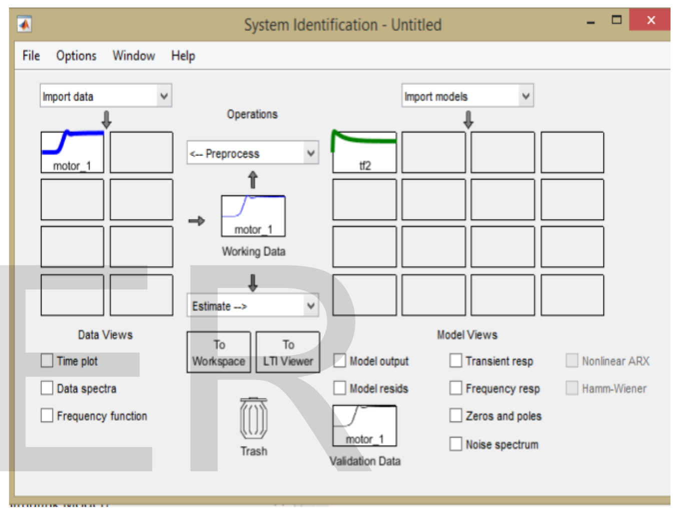

Fig: 6-1. MATLAB System Identification Toolbox's

The process of system identification needs a few steps in order to perform data collection. The first step to obtaining data is to measure in - time or frequency domain input and output signals from the system. In this case, the PID system frequency domain is used to boost the system. Model structure is selected by comparing system identification outcomes. The next step is to apply an estimated method value for adjustable parameters in the system model structure to be tested in Simulink from the system identification toolbox provided by MATLAB. The last step is to analyze the predictable model to asses if the model is adequate for the Quadrotor appliance requirement.

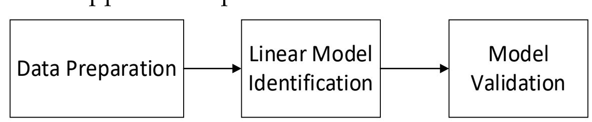

Fig: 6-2 Flow diagram of system identification

6.1 Simulation Results

The simulated results are described in the subsequent sections below

6.1.1 Motor Dynamics

In mostly used quadrotor motors, brushless DC motors are coupled with propellers. These engines generate high torque where the voltage of the input depends on the rotor speed [20].

$$\dot{\omega}=\frac{K_e v}{R\times\left(J_m+J_p\right)}-\frac{K_e^2\omega}{R\times\left(J_m+J_p\right)}-\frac{d\omega^2}{\left(J_m+J_p\right)}$$

In MATLAB, the motor dynamics are simulated:

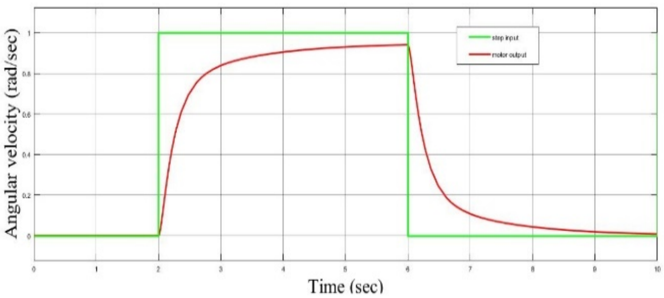

Fig: 6-3 DC motor Dynamic

The entire rotor dynamics have been identified and validated using the MATLAB Identification Toolbox. To reproduce the dynamics between the speed set - point of the propeller and its true speed, a first order transfer function is sufficient.

$$G(s)=\frac{15.5}{s+0.4805}$$

The above equation shows that the linearization of the initial condition of the model continuous time transfer functions.

6.1.2 Identification results

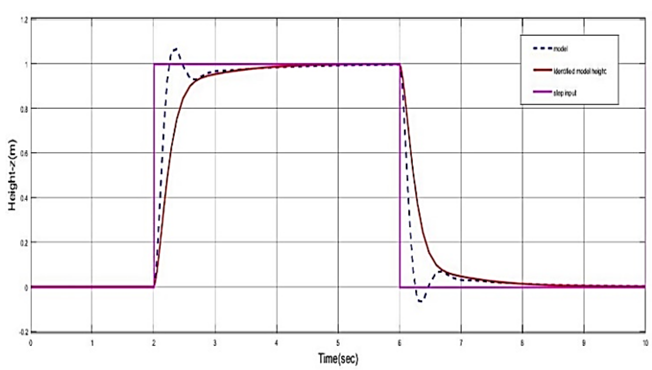

Fig:7

Fig: 6-4 Identification result for Height (Z)

Fig: 6-4 shows the identification and system input output results. Red line shows the Stimulated or predicted output of transfer function (t_{f1}). The value of best fit to generate the transfer function is 93.03% that had achieved the target for altitude control model.

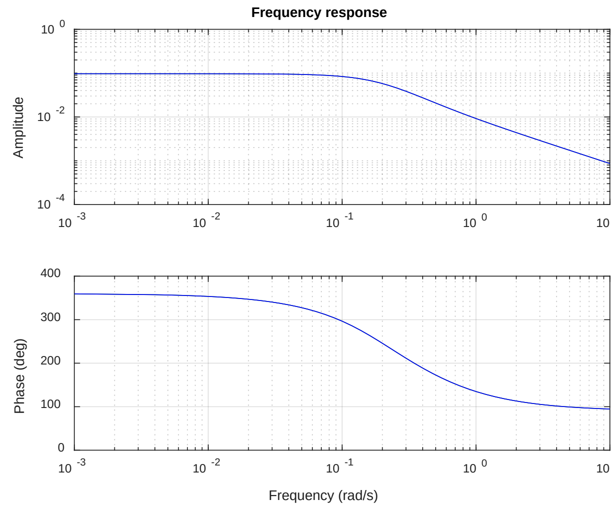

Fig: 6-5 Frequency response at hover condition (Z)

Fig: 6-5 shows the quadrotor's frequency response in the hover condition. The amplitude vs. frequency graph shows the diminishing amplitude pattern as the frequency increases.

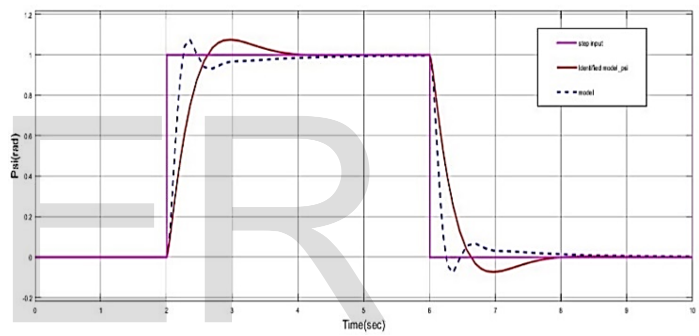

Fig: 6-6 Identification result for Pitch

6.1.3 Model Validation:

By double maneuvers selected while flight-testing time, the extracted models verified with the time-domain. In the system identification process, these maneuvers were not used during the extraction of the dynamic model. The inputs and initial conditions selected for the models, which identified the responses on all axis or directions. The responses of the simulation are then compared for the outcomes collected during the flight test. Fig: 6-8 Illustrate results of all these verifications. The dynamic models obtained perfectly compared the flight-test data.

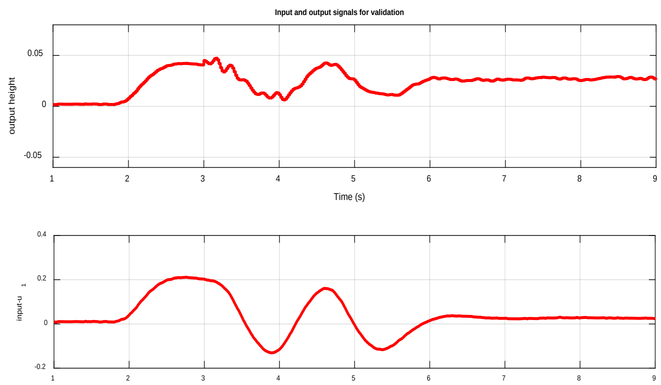

Fig: 6-7 Input and output signal for Validation

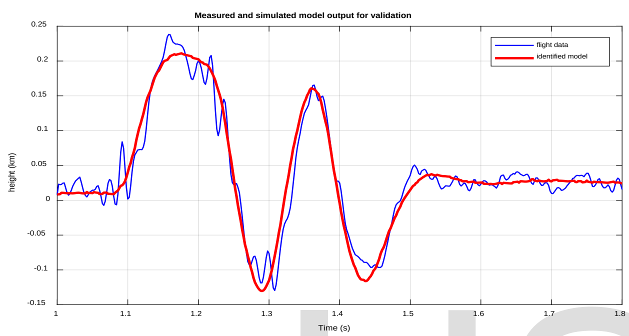

Fig: 6-8 Measured and simulated model for validation

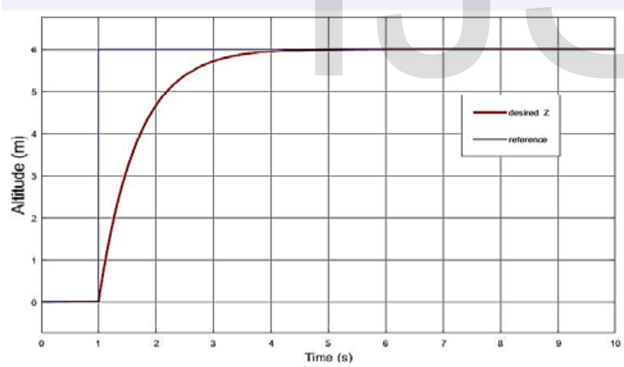

Fig: 6-9 Closed loop Simulation result

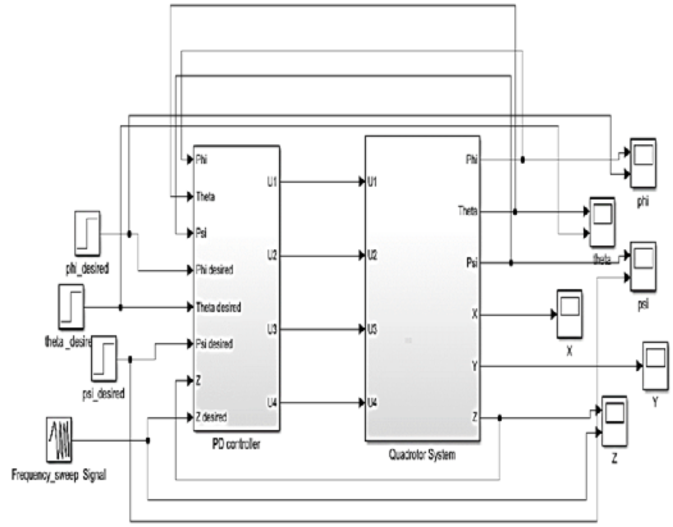

Fig: 6-10 A complete Simulink model of Quadrotor

7. CONCLUSION

This research focused on the identification and modeling of the hover quadrotor system. In order to identify the Quadrotor system in a frequency technique, a frequency sweep signal has been used to stimulate the Quadrotor. The model variables could then be extracted depending onto the flight test data and a validation was performed to validate the accuracy of system identification method and modeling.

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